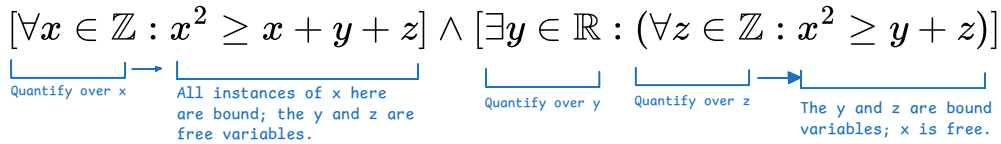


12 - Predicate logic (continued)

Free vs bound variable. A variable is bound if it is within the scope of a quantifier; otherwise it is free.



Convention. In common speech, a free variable is implicitly universally quantified:

Example. If $x \geq 1$, then $x^2 \leq x^3$. \longrightarrow For all $x \in \mathbb{R}$, if $x \geq 1$, then $x^2 \leq x^3$.
 $\forall x \in \mathbb{R} : [x \geq 1 \Rightarrow x^2 \leq x^3]$

$xy = 0 \Rightarrow x = 0 \text{ or } y = 0 \longrightarrow \forall (x, y) \in \mathbb{R}^2 : [xy = 0 \Rightarrow (x = 0) \vee (y = 0)]$

Negating quantifiers. Intuitively \forall and \exists are supersized versions of \wedge and \vee :

$$\forall x \in \{x_1, \dots, x_n\} : P(x) \equiv P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$$

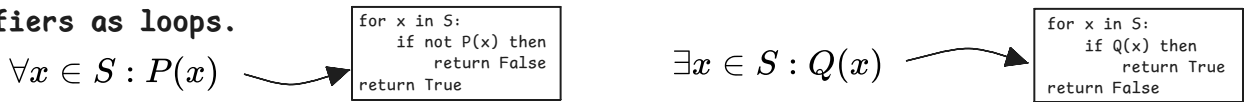
$$\exists x \in \{x_1, \dots, x_n\} : P(x) \equiv P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$$

So \neg distributes across terms and flips \wedge and \vee (de Morgan's law):

$$\neg[\forall x \in U : P(x)] \equiv \exists x \in U : \neg P(x)$$

$$\neg[\exists x \in U : P(x)] \equiv \forall x \in U : \neg P(x)$$

Quantifiers as loops.



Vacuous quantification.

$$\forall x \in \emptyset : P(x) \equiv T$$

$$\exists x \in \emptyset : P(x) \equiv F$$

Optional Homework due April 1 or 2.

Show your work. Answer without work receives no credit.

1. True or False?

- (a) $\exists x \in \mathbb{R} : x^2 > x$
- (b) $\exists x \in \mathbb{R} : x^2 = -1$
- (c) $\exists x \in \mathbb{R} : x^2 + 2 > 1$
- (d) $\forall x \in \mathbb{N} : (x^2 \neq x) \vee (x = 0) \vee (x = 1)$
- (e) $\exists n \in \mathbb{N} : n^2 \equiv 3 \pmod{4}$

2. Explain why this proposition is false: $\forall x \in \mathbb{R} : x^2 \geq x$

3. Let $C(x)$ ="x has a cat", $D(x)$ ="x has a dog", $F(x)$ ="x has a ferret.", S ={students in your class}. Formalize:

- (a) A student in your class has a cat, a dog, and a ferret.
- (b) All students in your class have a cat, a dog, or a ferret.
- (c) Some student in your class has a cat and a ferret but not a dog.
- (d) No student in this class has a cat, a dog, and a ferret.
- (e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals.